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AN EXPONENTIAL VOLTAGE CONTROLLED STATE VARIABLE FILTER

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OBJECTIVE

The purpose of this project is to design a flexible filter module for use in electronic music synthesizer applications. To achieve maximum versatility from this module, certain specifications should be met, which are as follows:

1. The filter should be able to pass the entire audio frequency range of 20Hz-20kHz. (Actually the response should go down to DC to allow processing of control functions)

2. The cutoff frequency should be variable over the range of 20Hz to at least 15kHz. (Control to very high frequencies is not necessary because the effect of filtering would not be audible)

3. The cutoff frequency should be voltage controlled, with an exponential characteristic of 1 volt per octave. I.E. the cutoff frequency will double for every volt of control voltage. There should be several inputs for control which are summed.

4. The 'Q' of the filter should be variable, although not necessarily voltage controlled. Values from under 1 to over 100 should be attainable.

5. There should be several signal inputs which are summed together for the audio.

6. The filter should provide highpass, bandpass, and lowpass outputs simultaneously, with an option for notch response if needed.

7. For music applications, the accuracy of the cutoff frequency with respect to the control voltage should be within 1% over any five octave range of frequencies.

8. Since most systems will require several of these filter modules, the cost of each should be as small as possible, in keeping with the above objectives.

DESIGN

Since it is desired to have simultaneous highpsss, bandpass and lowpass outputs, the state variable filter configuration was chosen as the basis for this design. Additional advantages of the state variable configuration include the fact that center frequency and 'Q' and gain are all independantly adjustable by varying resistors only. Also, very high 'Q' is easily attainable, and the sensitivity of the center frequency and 'Q' with respect to component tolerances. Of course in a filter where all parameters are adjustable, sensitivity isn't too important. Although it isn't really necessary for this design, the basic tranfer function for the lowpass output is as follows:

$$\frac{-\left(\frac{1}{R_7}\right)\left(\frac{1}{R_1 R_2 C_1 C_2}\right)\left[\frac{1+R_6/R_5}{1+R_3/R_4}\right]}{S^2 + S\left(\frac{1}{R_1 C_1}\right)\left[\frac{1+R_6/R_5}{1+R_4/R_3}\right] + \frac{R_6}{R_5}\left(\frac{1}{R_1 R_2 C_1 C_2}\right)} \qquad \text{where } R_6 = R_5 = R_7,\ R_1 = R_2,\ C_1 = C_2$$

The equations for cutoff frequency and 'Q' which are derived from this transfer function are what is necessary to design this filter:

$$f_c = \frac{1}{2\pi R_1 C_1} \quad \text{for } R_1 = R_2,\ C_1 = C_2,\ R_5 = R_6 \qquad Q = \frac{R_3 + R_4}{2R_3}$$

Since the state variable filter has been chosen as the basic configuration, it must now be modified to allow voltage control of cutoff frequency. The cutoff frequency can be varied by changing either the resistors or the capacitors of the two integrator stages, or some combinination of the two. The only voltage variable capacitors in existance are varactors, and they have a small range of variation, and a maximum capacitance of several picofarads, making them unusable for this design. Therefore some kind of voltage controlled resistor must be created. Possible approaches using FET's or LED-photocell pairs have been tried, but there resistance vs. input voltage is highly nonlinear and difficult to compensate for. Also FET's have a very limited signal handling capacity, and photocells have very slow response. Fortunately a new type of op-amp has been developed, called the operational transconductance amplifier.

The operational transconductance amplifier, or OTA, is a new device which approximates a voltage controlled current source with current controlled transconductance. The device has a high impedance bi-directional current source output instead of a low impedance voltage source output, but otherwise has all other characteristics of an ideal op-amp.

The CA3080 OTA manufactured by RCA is the only device of its type available at the present time. It is an interesting chip, since it uses no resistors at all, only transistors and diodes. All currents are set by the control input, Iabc, (for amplifier bias current). This causes extremely fast destruction of an OTA if the Iabc terminal is connected to any voltage source without current limiting resistance. On the good side, every parameter of the op-amp is controlled by this current, most notably the transconductance, gm. For the CA3080, gm = 19.2 Iabc. This relation is linear for Iabc values from .1uA to 600uA, nearly a five decade range,or about 16 octaves.

The use of the OTA as a voltage variable resistor comes from its current source output. A resistor connected from a voltage V to the inverting input of an op-amp which is at virtual ground is essentialy sourcing current of the value V/R. Why not replace that fixed current source with a variable one, namely the output of a CA3080? Doing so results in a current controlled effective resistor whose value can be programmed by a voltage divider at the input of the OTA. (see appendix for calculations)

The only task remaining in the design is that of creating a current source to drive the Iabc inputs of the OTAs. The basic exponential converter has a voltage output and uses NPN transistors, with Q2's collector sinking current, causing the following op-amp to have a positive output. This positive output voltage could be passed through resistors and into the Iabc inputs of the CA3080's. Unfortunately, the Iabc input is referenced to V-, and to have zero current flow, the output of the driving op amp would have to go to V-, which is impossible for normal op-amps. There are ways around that problem, but it is just a bad idea to try and accurately vary a voltage over five decades. If the maximum is taken to be 10 volts, then the minimum has to be .0001 volts, well below the input ofset voltage of ordinary op-amps. Therefore the exponential control current must be generated as a current and remain a current.

In the basic exponential converter circuit the collector of Q2 acts as a current sink of a current proportional to the exponential output voltage. If the logging transistors are replaced by PNP transistors, and the reference voltage is made negative, all polarities in the circuit are reversed, (except the power supplies) and an exponential current output is available, saving an op-amp in the process. The basic exponential circuit is set up to exponentiate to the base e. A transformation using the identity A**x = e**x(lnA) is included in the input voltage divider (see appendix for calculations) to give a base of 2 for octave response.

The output current of the exponential converter must supply the Iabc inputs of 2 CA3080s. It would be nice if each OTA could have its own collector to drive it but dual matched monolithic PNP transistors are hard enough to find, let alone triples. So the single output current must be accurately divided into halves. The easiest way to do this is to use two precision resistors of equal value. A voltage drop does appear accross these resistors, but the total current isn't affected, providing the Ic vs. Vc characteristics of the transistor are relativly flat. These resistors also serve to protect the OTAs in the event of a catastrophic failure of the exponential converter. The effect of mismatch between these dividing resistors is not important, since if one OTA recieved a little extra current, the other would recieve the same amount less. The same relation holds for the equivilant resistances, and since the resistor values are multiplied together to determine the cutoff frequency, the error would cancel out. The mismatch would tend to reduce the maximum attainable 'Q' though.

Since the control input results from the summation of several inputs, it is possible for the sum to become greater than the voltage required to produce the maximum cutoff frequency. This condition could cause excess Iabc current to flow in the OTAs, so some type of limiting needs to be provided. The output voltage of the op-amp in the exponential converter circuit is proportional to the exponential output current. A zener diode can be added from the output to the inverting input of the op-amp to limit the output current to any value. The chosen limit point is 600ua, corresponding to a frequency of about 16kHz.

The choice of op-amps for this circuit is straightforward: use the cheapest ones you can get away with. The OTAs leave little choice, CA3080 is the only possibility. The op-amps the CA3080s drive are used as integrators, and therefore should have low input current. The LM308 FET input op-amp was chosen for this application. The summing stage at the input of the filter poses no special requirements, so a 741 was used. The exponential converter is a fairly critical circuit, and the required bandwidth is not high, so a LM307 was chosen due to its low input current and temperature stability. (FET input op-amps have very poor DC temperature stability, but here they are used in an AC application) Precision resistors are used in the exponential converter at critical locations such as the 1 volt per octave input, and the current divider. Two trimpots are used, one for 1 V/oct trimming and the other for base frequency (Vc=0) set.

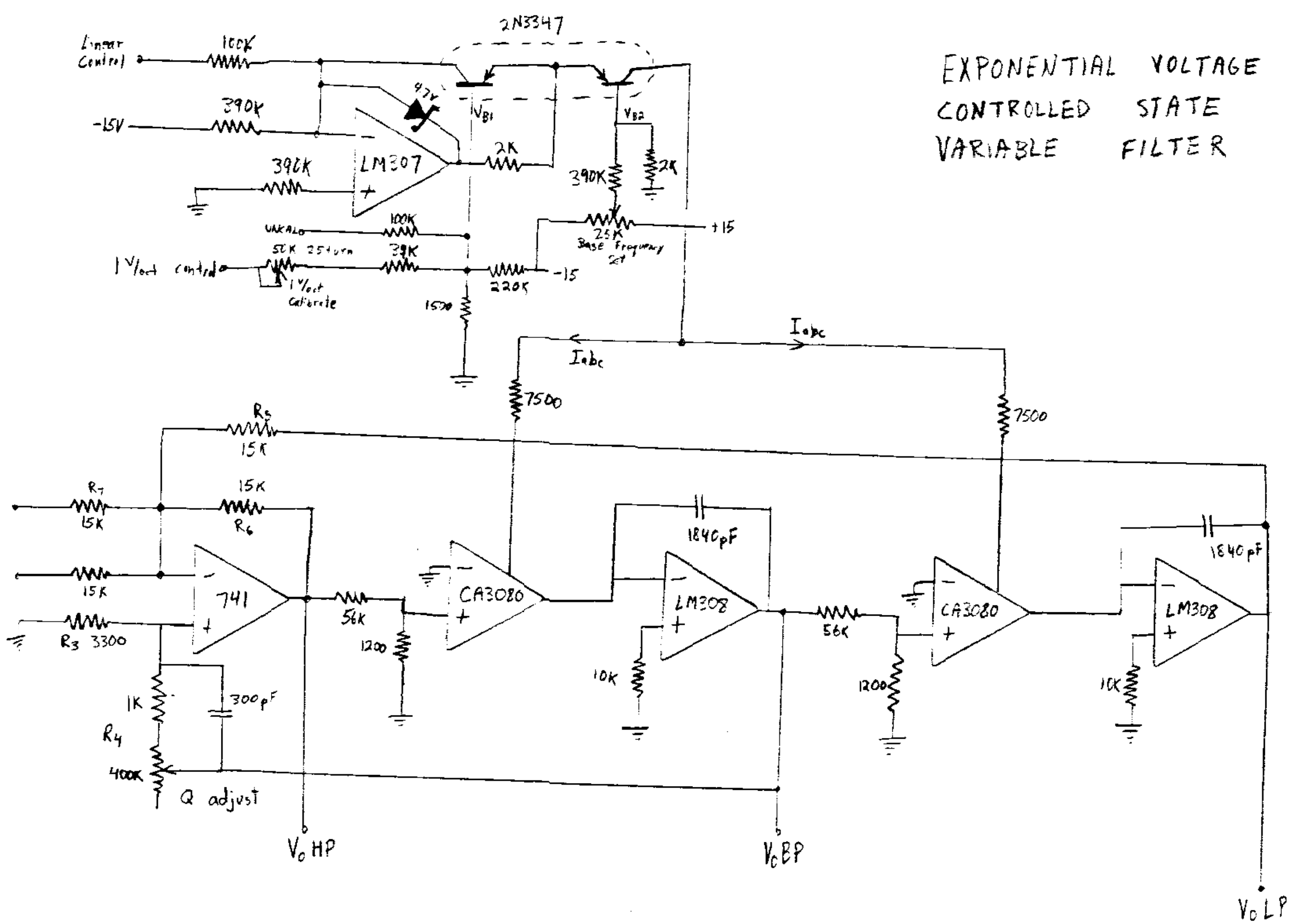
EXPERIMENTAL RESULTS

In initial testing, several problems were encountered. First and most unpleasent, the PC board was exposed with the copper on the wrong side, requiring each op-amp to have its leads bent over to the opposite side of the package in order to get them to go on the board. While the filter was being tested over speakers, it was discovered that it would go into oscillation at high Q and high cutoff frequency conditions. A 30 volt p-p signal at 10 KHz applied to an amplifier set to amplify 100 millivolt signals to comfortable listening levels is an experience not worth repeating. A capacitor added across the Q control to reduce the Q at high frequencies solved the problem. The oscillation was due to the fact that the filter has constant bandwidth for various cutoff frequencies, so as the frequency is increased, the Q increased also. Compounding this affect, the op-amps contribute additional phase shift to the loop as frequency is increased.

Another affect that popped up late in the testing process was a difference in the passband gains between the highpass and lowpass outputs. No explanation has been discovered, but the effect is small and makes no difference in the intended use of the module.

The overall performance substantially exceeds specifications. The voltage controlled frequency range extends from the limiter imposed maximum of 16KHz to a low that is to low to accurately measure. As an indication of the frequency, when the control voltage was zero and the signal input was disconnected, the trace on the scope slowly bounced up and down in a decaying fashion, sugesting that the resonance point is well under one hertz. The DC ofset at the output was measured and is under .1 volts for the frequency range of 20hz to 16KHz, rising to about .5 V at minimum frequency. The highpass frequency response extends up to about 1MHZ, which is to be expected for a 741 operating with a gain of 1. The circuit limits Q to a minimum of .66 and the maximum is too high to be measured, due to a tendency of the filter to 'lock on' to the input as resonance is approached and not release until resonance is well passed. The maximum Q is at least several hundred. The exponential converter was sucessfully calibrated to precise 1 volt per octave response. The filter was able to resonate musical notes at each octave with integer voltage variations on the control input.

EXPONENTIAL VOLTAGE CONTROLLED STATE VARIABLE FILTER



CONCLUSIONS

The overall conclusion is that this filter is the ideal general purpose building block required for music synthesis and other applications. It meets its specifications, fulfills its objectives, and is very low in cost compared to anything at all similar. The actual cost is around $8.ØØ.

There are several possible improvements that need to be explored. This prototype didn't have temperature compensation due the lack of a positive temperature coefficiant thermistor. A possible alternative is adding an additional op-amp input stage to the exponential converter with the thermistor in the feedback loop. This would have the added advantage of eliminating the error due to the base current of Q1 from the divider, and providing a true summing node rather than the present approximation. Also, better high frequency performance can be obtained by using feedforward compensation on the LM3Ø8 integrators, which will boost their gain bandwidth product to 3 MHZ. Another op-amp could be added to sum highpass and lowpass outputs in various proportions for various notch functions.

In conclusion, this project has been worthwhile, resulting in the creation of a genuinely useful circuit that has many applications.

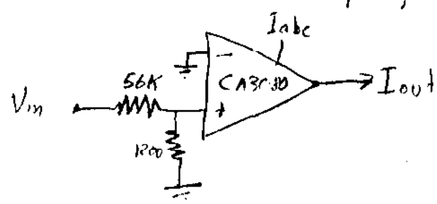
CALCULATION OF FILTER Components

For State Variable Filter:

$$f_c = \frac{1}{2\pi RC}$$

For OTA $\quad g_m = 19.2\ I_{abc}$

Select OTA input voltage divider to limit 5V max signal to 100mV at input, a ratio of .02



For this circuit $I_{out} = \frac{1200}{1200+56K}(19.2)\, I_{abc}\, V_{in}$

$R_{equiv} = \frac{V_{in}}{I_{out}} \quad \frac{56K+1200}{(19.2)(1200)I_{abc}} = \frac{2.48}{I_{abc}}$

Substituting into f_c, $\quad f_c = \frac{I_{abc}}{2.48\ C\ (2\pi)}$

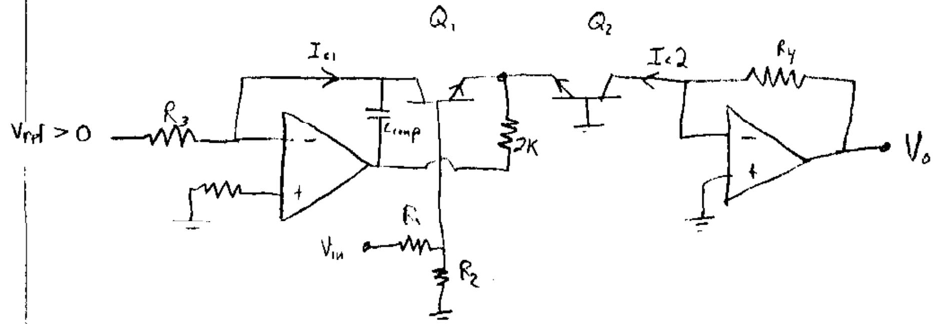
To achieve $20hz \le f_c \le 15\ kHz$ with $.5\mu A \le I_{abc} \le 500\mu A$

choose $\quad C = 1840\ pF$

then $\quad f_c = \frac{I_{abc}}{(2.87 \times 10^{-8})} = (34.8 \times 10^{6})\, I_{abc}$

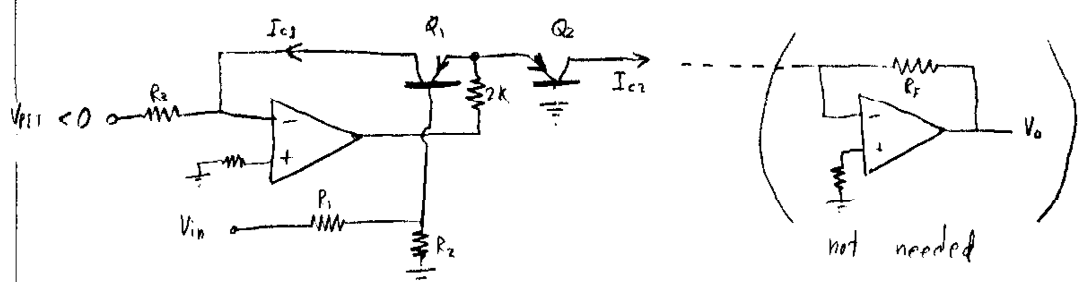
choose $R_5, R_6, R_7 = 15K$ since I have a lot of 15K 2% Resistors

EXPONENTIAL CONVERTER CALCULATIONS



BASIC EXPONENTIAL CONVERTER

$$V_o = \frac{V_{REF}}{R_3 R_4} e^{\left(\frac{-q}{KT} \frac{R_2}{R_1+R_2} V_{in}\right)}$$



TRANSFORMED FOR REVERSE CURRENTS

for a silicon transistor $V_{BE} = \frac{KT}{q} \ln\left(\frac{I_c}{I_0}\right)$
I_{c1} is constant due to the op-amp
$I_{c1} = \frac{V_{REF}}{R_3}$

So V_{BE1} is constant: $V_{BE1} = \frac{KT}{q} \ln\left(\frac{V_{REF}}{R_3 I_0}\right)$

When the base voltage of Q_1 is varied by an input voltage, the op-amp will maintain V_{BE1} constant by varying the common emmitter voltage

$$V_e = V_{B1} + V_{BE1}$$

assume $V_{B2} \neq 0$ (actually it is slightly variable as a base level trim)
Then, for Q_2, $V_{BE} = V_e - V_{B2} = V_{B1} + V_{BE1} - V_{B2}$

For Q_2 $V_{BE} = V_{B1} - V_{B2} + \frac{KT}{q} \ln\left(\frac{V_{REF}}{R_3 I_0}\right)$

$V_{BE} = \frac{KT}{q}\left(\ln\left(\frac{I_c}{I_0}\right)\right) \rightarrow \frac{q}{KT} V_{BE} - \ln\left(\frac{I_c}{I_0}\right) \rightarrow \exp\left(\frac{q}{KT} V_{BE}\right) = I_c/I_0 \quad I_c = I_0 \exp\left(\frac{q}{KT} V_{BE}\right)$

$I_c = I_0 \exp\left[\frac{q}{KT}\left\{V_{B1} - V_{B2} + \frac{KT}{q} \ln\left(\frac{V_{ref}}{R_3 I_0}\right)\right\}\right] = I_0 \exp\left[\frac{q}{KT}(V_{B1} - V_{B2}) + \ln \frac{V_{REF}}{R_3 I_0}\right]$

$I_c = I_0 \exp\left[\frac{q}{KT}(V_{B1} - V_{B2})\right] \exp\left[\ln \frac{V_{REF}}{R_3 I_0}\right] = \frac{V_{REF}}{R_3} \exp\left[\frac{q}{KT}(V_{B1} - V_{B2})\right]$

From Previous Calculations the following two relations result

$f_c = (34.8 \times 10^6) I_{abc}$ $\qquad$ $I_c = \frac{V_{REF}}{R_3} \exp\left[\frac{q}{KT}(V_{B1} - V_{B2})\right]$

Due to current divider

$I_{abc} = \frac{1}{2} I_c$

$f_c = 17.4 \times 10^6 I_c$

Choose $V_{REF} = -15$ naturally
let $R_3 = 390\,K$ to put $I_{c1} = 38.5\,\mu A$, near the center of the range of I_{c2} for best matching over entire Range

$I_c = 38.5 \times 10^{-6} \exp\left(\frac{q}{KT}(V_{B1} - V_{B2})\right)$

$f_c = (17.4 \times 10^6)(38.5 \times 10^{-6}) \exp\left[\frac{q}{KT}(V_{B1} - V_{B2})\right]$

$f_c = 670 \exp\left[\frac{q}{KT}(V_{B1} - V_{B2})\right]$ $\qquad$ now let $V_{B2} = 0$ so trimpot can ideally be at center of range

$f_c = 670 \exp\left[\frac{q}{KT} V_{B1}\right]$

Desired Response: $f_c = (16)(2^{V_{in}})$ $\qquad$ 16 hz is base frequency when $V_{in} = 0$

using the identity $A^x = e^{x \ln A}$

$$f_c = 16\, e^{V_{in} \ln 2}$$

Equate desired and actual response functions to get a function for V_{B1} in terms of V_{in}

$$16\, e^{V_{in} \ln 2} = 670 \exp\left[\frac{q}{KT} V_{B1}\right]$$
$$e^{V_{in} \ln 2} = 41.88 \exp\left[\frac{q}{KT} V_{B1}\right]$$
$$e^{V_{in} \ln 2} = e^{3.73} e^{q/KT (V_{B1})}$$
$$e^{V_{in} \ln 2} = e^{\frac{q V_{B1}}{KT} + 3.73}$$

$\ln 41.88 = 3.73$; $e^{3.73} = 41.88$

$$V_{in} \ln 2 = \frac{q V_{B1}}{KT} + 3.73$$
$$V_{in} \ln 2 - 3.73 = \frac{q}{KT} V_{B1}$$
$$V_{B1} = \frac{KT \ln 2}{q}(V_{in}) - 3.73 \frac{KT}{q}$$

$\frac{K}{q} = 8.63 \times 10^{-5}$

$$V_{B1} = (5.98 \times 10^{-5}) T V_{in} - (3.22 \times 10^{-4}) T$$

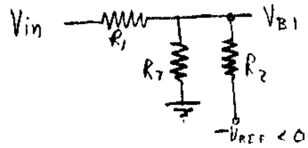
T in Kelvin

This can be viewed as a function of the form $Y = MX - B$ where $M = (5.98 \times 10^{-5}) T$ and $B = (3.22 \times 10^{-4}) T$

at Room temp $T = 300$

$M = 1.79 \times 10^{-2}$ $\qquad$ $B = 9.66 \times 10^{-2}$

The following circuit will be used to implement the function



Since the gain of Q_1 is high, and relatively low impedances will be chosen for R_T, assume $I_{B1} \approx 0$

$$\frac{V_{in} - V_{B1}}{R_1} = \frac{V_{B1}}{R_T} + \frac{V_{B1} + V_{REF}}{R_2}$$

$$R_T R_2 V_{in} - R_T R_2 V_{B1} = R_1 R_2 V_{B1} + R_1 R_T V_{B1} + R_1 R_T V_{REF}$$

$$V_{B1} = \frac{V_{in} \; R_T R_2}{R_2 R_T + R_1 R_2 + R_1 R_T} - \frac{V_{REF} \; R_1 R_T}{R_2 R_T + R_1 R_2 + R_1 R_T}$$

This is also a function of the form $Y = MX + B$

Equate the two M values and the two B values and solve for R_1 and R_2 given a R_T

continued

Calculation of Input Network
by solving for R_1 and R_2

$$\frac{R_2 R_T}{R_2 R_T + R_1 R_2 + R_1 R_T} = M$$

$$R_2 R_T = M[R_2 R_T + R_1 R_2 + R_1 R_T]$$

$$R_2 R_T = M R_2 (R_T + R_1) + M R_1 R_T$$

$$R_2 R_T - M R_2 (R_T + R_1) = M R_1 R_T$$

$$R_2 (R_T - M(R_T + R_1)) = M R_1 R_T$$

$$R_2 = \frac{M R_1 R_T}{R_T - M(R_T + R_1)}$$

$$B = \frac{N R_1 R_T}{R_2 R_T + R_1 R_2 + R_1 R_T}$$

$$= \frac{N R_1 R_T}{R_2(R_T + R_1) + R_1 R_T}$$

$R_1 = X \quad R_2 = Y \quad R_T = Z$

$$Y = \frac{MXZ}{Z - M(Z+X)}$$

$$B = \frac{NXZ}{Y(Z+X) + XZ}$$

$$B = \frac{NXZ}{\frac{MXZ(Z+X)}{Z - M(Z+X)} + XZ} \left(\frac{Z - M(Z+X)}{Z - M(Z+X)}\right)$$

$$B = \frac{NXZ(Z - M(Z+X))}{MXZ(Z+X) + XZ(Z - M(Z+X))}$$

$$B = \frac{N(Z - M(Z+X))}{M(Z+X) + (Z - M(Z+X))}$$

$$= \frac{N(Z - MZ - MX)}{MZ + MX + Z - MZ - MX}$$

$$= \frac{NZ - NMZ - NMX}{Z}$$

$$B = \frac{N(Z - M(Z+X))}{Z}$$

$$B = \frac{NZ}{Z} - \frac{NMZ}{Z} - \frac{NMX}{Z}$$

$$= N - NM - \frac{NMX}{Z}$$

$$\frac{NMX}{Z} = N - NM - B$$

$$NMX = ZN - ZNM - BZ$$

$$X = \frac{ZN}{NM} - \frac{ZNM}{MN} - \frac{BZ}{NM}$$

$$= \frac{Z}{M} - Z - \frac{BZ}{NM}$$

$$X = Z\left(\frac{1}{M} - 1 - \frac{B}{NM}\right)$$

Transforming to Original Variables

$$R_1 = R_T\left(\frac{1}{M} - 1 - \frac{B}{V_{REF}\, M}\right)$$

$$R_2 = R_T\left(\frac{M R_1}{R_T - M(R_T + R_1)}\right)$$

Inserting the values of M and B from
the V_{B1} to V_{in} relation: (using T = 300K, room temp) ($V_{REF} = 15$)

$$R_1 = R_T\left(\frac{1}{1.79\times10^{-2}} - 1 - \frac{9.66\times10^{-2}}{(15)(1.79)\times10^{-2}}\right)$$

$$R_1 = 54.4\ R_T$$

$$R_2 = R_T\left(\frac{(1.79\times10^{-2})(54.4R_T)}{R_T - 1.79\times10^{-2}(R_T + 54.4R_T)}\right)$$

$$R_2 = R_T\left(\frac{(1.79\times10^{-2})(54.4)}{1 - 1.79\times10^{-2}(55.4)}\right)$$

$$R_2 = 151\ R_T$$

Choosing $R_T = 1500\ \Omega$

$$\boxed{\begin{array}{l} R_1 = 22K \\ R_2 = 220K \end{array}}$$

trim for 1 V/oc -
trim for base freq = 16
(or use 1/ez adjustment)

Thermal compensation is achieved by substituting various values of T into the M and B equations and re calculating R_1 and R_2 for each case. Then a thermistor can be found to replace R_T which will allow R_1 and R_2 to be constant. It turns out that the thermistor must have a positive temperature coefficient.

Calculation of Pole Positions

Denominator of transfer function

$$s^2 + s\,\frac{1}{R_1C_1}\left[\frac{1 + R_6/R_5}{1 + R_4/R_3}\right] + \frac{R_6}{R_5}\left(\frac{1}{R_1R_2C_1C_2}\right)$$

$R_6 = R_5$

$C_1 = C_2 = C$

let $R_4 = R_3$ $(Q = 1)$ for this calculation

$$R_1 = R_2 = R_{eq} = \frac{2.48}{I_{abc}} = \frac{4.96}{I_c} = \frac{4.96}{(16)2^{V_{in}}}$$

$$R = \frac{1}{(3.22)(2^{V_{in}})}$$

$$s^2 + s\left(\frac{(3.22)(2^{V_{in}})}{C}\right) + \left[\frac{(3.22)2^{V_{in}}}{C}\right]^2$$

$$s^2 + As + A^2$$

$$s = \frac{-A}{2} \pm \frac{j\sqrt{3}}{2}A$$

for $C = 1840\times10^{-12}$ $\quad A = 1.753\times10^9\ 2^{V_{in}}$